Parametrization of Tachyon Field

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Abstract

We assume that universe is dominated by non-relativistic matter and tachyon field and reconstruct the potential of tachyon field directly from the effective equation of state (EOS) of dark energy. We apply the method to four known parametrization of equation of state and discuss the general features of the resulting potentials.

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I. INTRODUCTION

Observations of Type Ia supernovae indicate that our universe has entered a phase of accelerated expansion in the recent past.[1,2] The accelerated expansion has been attributed to the existence of mysterious dark energy with negative pressure. At present, there have been many papers devoted to addressing the nature of the dark energy. The cosmological constant Λ is the simplest and most obvious candidate for dark energy. However this model suffers from the theoretical problems—fine-tuning and coincidence problems. These problems have led to a variety of alternative models where the dark energy component varies with time, such as Quintessence,[3,4] Chaplygin gas,[5,6] modified gravity,[7] phantom,[8] K-essence[9] and so on. For the models of dark energy with a scalar field (eg. quintessence, phantom, K-essence), one can design many kinds of potentials and then study equation of state (EOS) for the dark energy. On the other hand, the potential can also be reconstructed from a parametrization of the EOS fitting the observational data.[10] The latter has the advantage that it does not depend on a specified model of dark energy and, therefore, is also called a model-independent method.[11]

Recently it has been suggested that rolling tachyon condensates, in a class of string theories, may have interesting cosmological consequences. In this article, we concentrate on the issue of the tachyon as a source of the dark energy. The tachyon[12,13] is an unstable field which has become important in string theory through its role in the Dirac-Born-Infeld (DBI) action which is used to describe the D-brane action.[14–17] A number of authors have already demonstrated that the tachyon could play a useful role in cosmology,[18–21] independent of the fact that it is an unstable field. The tachyon can act as the source of dark energy depending on the form of the associated potential.[22–25] The purpose of this paper is to use the model-independent method to reconstruct the potential of tachyon field from the EOS of the dark energy.

Various parametrization of the EOS of dark energy has been presented and investigated. In this paper, we will use four of them[26–30] to reconstruct the tachyon potential $V(T)$ directly from the EOS $w_T(z)$ and then discuss the general features of the resulting potentials. Moreover the difference between tachyon and quintessence is also obtained for the evolution of the potential. The outline of this paper is as follows: In section II, we introduce the DBI action and the associated equation of motion for the tachyon field. Furthermore, the potential of tachyon field is reconstructed from EOS of dark energy $w_T(z)$. Section III we apply the model independent method to the four typical parametrizations of EOS and discuss the general features of the resulting potentials. Section IV contains the conclusions.

II. RECONSTRUCTING THE POTENTIAL OF TACHYON FIELD

The Dirac-Born-Infeld (DBI) type effective 4-dimensional action is described by[14,15]

$$S = \int d^4x \Big[\sqrt{-g}\frac{R}{2} - V(T) \times \sqrt{-det(g_{\mu\nu} + \partial_\mu T \partial_\nu T)} \Big], \tag{1}$$

where $V(T)$ is the potential of tachyon field T. The above DBI action is believed to describe the physics of tachyon condensation for all values of T as long as string coupling and the second derivative of T are small.

We consider a cosmological scenario in which the system is filled with non-relativistic matter and the tachyon field T. In a flat FRW metric, the density ρ_T and pressure p_T of tachyon field are given by

$$\rho_T = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}, \tag{2}$$

$$p_T = -V(T)\sqrt{1 - \dot{T}^2}. \tag{3}$$

The EOS of the dark energy is

$$w_T \equiv \frac{p_T}{\rho_T} = \dot{T}^2 - 1. \tag{4}$$

The Friedmann equation can be written as:

$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_T), \tag{5}$$

where ρ_M is density of non-relativistic matter.

For a spatially homogeneous tachyon field, the equation of motion is

$$\dot{\rho_T} + 3H(\rho_T + p_T) = 0, \tag{6}$$

which yield

$$\rho_T = \rho_{T0} \exp\Big[3\int_0^z (1 + w_T) d\ln(1+z)\Big], \tag{7}$$

where the dot denotes the differentiation with respect to t and subscript 0 represents the value of a quantity at present ($z = 0$). The redshift z is given by $1 + z = a_0/a$. From Eqs.(2),(4) and (7), we obtain

$$V[T(z)] = \rho_{T0}\sqrt{-w_T}\exp\Big[3\int_0^z (1 + w_T) d\ln(1+z)\Big], \tag{8}$$

$$\left(\frac{dT}{dz}\right)^2 = \frac{1+w_T}{(1+z)^2 H^2(z)}, \tag{9}$$

where $dz/dt = -(1+z)H(z)$. With the help of $\rho_M = \rho_{M_0}(1+z)^3$ and Eq.(7), the Friedmann equation (5) becomes

$$H(z) = H_0\Big[\Omega_{M0}(1+z)^3 + \Omega_{T0}\exp\Big[3\int_0^z (1+w_T)d\ln(1+z)\Big]\Big]^{\frac{1}{2}}, \tag{10}$$

where $\Omega_{M0} = \rho_{M0}/(3H_0^2/8\pi G), \Omega_{T0} = \rho_{T0}/(3H_0^2/8\pi G)$ and $\Omega_{M0} + \Omega_{T0} = 1$. Substituting into Eq.(9), we have

$$\frac{dT}{dz} = \pm\frac{\sqrt{1+w_T}}{H_0(1+z)\Big[\Omega_{M0}(1+z)^3 + (1-\Omega_{M0})\exp\Big[3\int_0^z (1+w_T)d\ln(1+z)\Big]\Big]^{\frac{1}{2}}}, \tag{11}$$

where the upper (lower) sign applies if $\dot{T} < 0(\dot{T} > 0)$. As the sign can be changed by the field redefinition, $T \to -T$, it is arbitrary. Thus we choose the lower sign in the following sections. Eqs.(8) and (11) are the potential and field function of tachyon field which we have reconstructed.

We define the following quantities

$$\tilde{V}[T(z)] \equiv \frac{V[T(z)]}{\rho_{T0}}, \quad \tilde{T}(z) \equiv \frac{T(z)}{H_0^{-1}}. \tag{12}$$

Eqs.(8) and (11) can be written as

$$\tilde{V}[T(z)] = \sqrt{-w_T}\exp\Big[3\int_0^z (1+w_T)d\ln(1+z)\Big], \tag{13}$$

$$\frac{d\tilde{T}}{dz} = \pm\frac{\sqrt{1+w_T}}{(1+z)\Big[\Omega_{M0}(1+z)^3 + (1-\Omega_{M0})\exp\Big[3\int_0^z (1+w_T)d\ln(1+z)\Big]\Big]^{\frac{1}{2}}}. \tag{14}$$

III. PARAMETRIZATIONS OF THE POTENTIAL $V(T)$

Now, let us discuss the evolutions of potential $V(T)$ and field function T of tachyon field. It was shown in Ref.10 that the quintessence potentials can be reconstructed from a given EOS of dark energy $w_\phi(z)$. In this spirit, we can also reconstruct the potential of tachyon field from a given concrete form of EOS $w_T(z)$. The following four cases are considered: a constant EOS parameter and the other three two-parameter parametrizations.

Case I: $w_T = w_0$ (Ref.26). For this case, w_T is a constant and Eqs.(13) and (14) can be written by

$$\tilde{V}(z) = \sqrt{-w_0}(1+z)^{3(1+w_0)}, \tag{15}$$

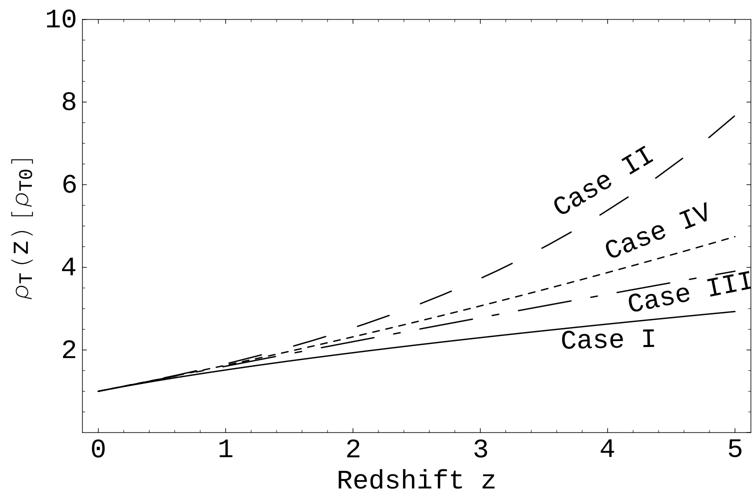

FIG. 1: Evolution of the energy density of tachyon $\rho_T(z)$.

$$\frac{d\tilde{T}}{dz} = -\frac{\sqrt{1+w_0}}{(1+z)\Big[\Omega_{M0}(1+z)^3 + (1-\Omega_{M0})(1+z)^{3(1+w_0)}\Big]^{\frac{1}{2}}}. \tag{16}$$

Case II: $w_T = w_0 + w_1 z$ (Ref.27). For this case, Eqs.(13) and (14) can be given by

$$\tilde{V}(z) = \sqrt{-w_0 - w_1 z}(1+z)^{3(1+w_0-w_1)} e^{3w_1 z}, \tag{17}$$

$$\frac{d\tilde{T}}{dz} = -\frac{\sqrt{1+w_0+w_1 z}}{(1+z)\Big[\Omega_{M0}(1+z)^3 + (1-\Omega_{M0})(1+z)^{3(1+w_0-w_1)} e^{3w_1 z}\Big]^{\frac{1}{2}}}. \tag{18}$$

Case III: $w_T = w_0 + w_1 \frac{z}{1+z}$ (Ref.28 and 29). Similar to Cases I and II, for this case we have

$$\tilde{V}(z) = \sqrt{-w_0 - w_1 \frac{z}{1+z}}(1+z)^{3(1+w_0+w_1)} e^{-3\frac{w_1 z}{1+z}}, \tag{19}$$

$$\frac{d\tilde{T}}{dz} = -\frac{\sqrt{1+w_0+w_1 \frac{z}{1+z}}}{(1+z)\Big[\Omega_{M0}(1+z)^3 + (1-\Omega_{M0}) e^{-3\frac{w_1 z}{1+z}}(1+z)^{3(1+w_0+w_1)}\Big]^{\frac{1}{2}}}. \tag{20}$$

Case IV: $w_T = w_0 + w_1 \ln(1+z)$ (Ref.30). For this case, Eqs.(13) and (14) can also be written

$$\tilde{V}(z) = \sqrt{-w_0 - w_1 \ln(1+z)}(1+z)^{3(1+w_0)+\frac{3}{2}w_1 \ln(1+z)}, \tag{21}$$

$$\frac{d\tilde{T}}{dz} = -\frac{\sqrt{1+w_0+w_1 \ln(1+z)}}{(1+z)\Big[\Omega_{M0}(1+z)^3 + (1-\Omega_{M0})(1+z)^{3(1+w_0)+\frac{3}{2}w_1 \ln(1+z)}\Big]^{\frac{1}{2}}}, \tag{22}$$

We have numerically evaluated the above equations. For the specific reconstruction we choose $w_0 = -0.8$, $w_1 = 0.1$, $\Omega_{M0} = 0.25$. Fig.1 and fig.2 show the evolutions of the energy

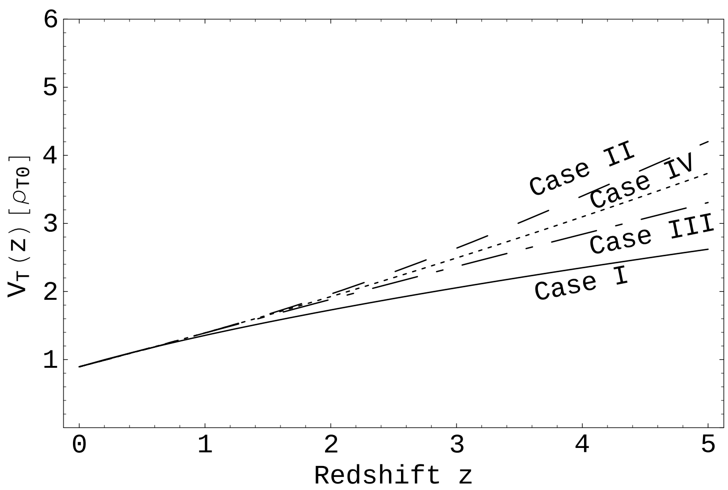

FIG. 2: Evolution of the potential of tachyon $V_T(z)$.

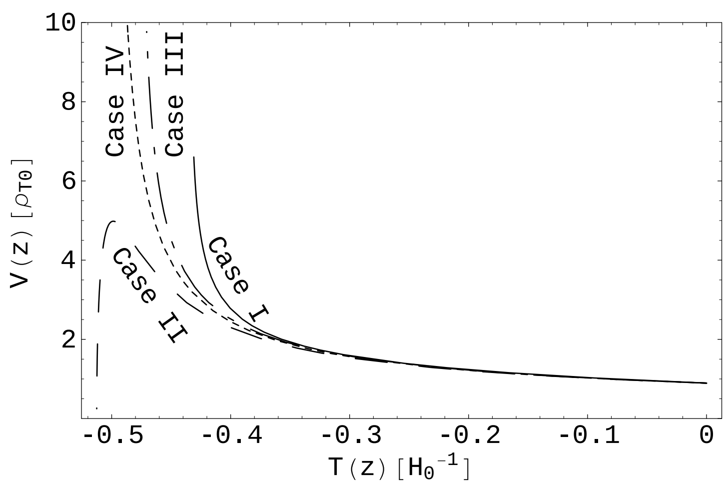

FIG. 3: Constructed tachyon potential $V(T)$.

density $\rho_T(z)$ and potential $V_T(z)$ of the tachyon with redshift z, respectively. At low redshift region, all models share the same asymptotical behavior, but deviate from this at redshift $z > 1$. Furthermore, the relation of the reconstructed potential $V(T)$ and field function T is showed in fig.3. We see that the four cases possess the same asymptotic behavior for the region $T > -0.3$ and deviate from this at $T < -0.3$. Moreover from fig.2 and 3, we note that when z decreases, the potential $V(T)$ decreases and T increases. This means that the potential $V(T)$ decreases as the universe expands. In addition we find there is potential hill for the evolution of tachyon potential in Case II, which is different from other three cases.

IV. CONCLUSIONS

In this paper, we have considered a spatially flat FRW universe which is dominated by the non-relativistic matter and a spatially homogeneous tachyon field T. By introducing Dirac-Born-Infeld (DBI) action, we have obtained the equation of motion for tachyon field. Then we have used the model-independent method to reconstruct the potential of tachyon field from EOS of dark energy $w_T(z)$. Furthermore, for four known EOS of dark energy we have plotted the evolutions of the potential $V(T)$ and the energy density $\rho_T(z)$. By analysis, we have given the following results:

- The potential $V_T(z)$ and energy density $\rho_T(z)$ for the four cases share the same asymptotical behavior at low redshift ($0 < z < 1$) and deviate from this at redshift $z > 1$.

- When EOS parameter for dark energy $w_T(z) = w_0 + w_1 z$ (Case II), there is potential hill for the evolution of tachyon potential, which is the different from the other three cases.

- The reconstructed tachyon potential $V(T)$ is in the form of a runaway potential at $T > -0.5$.

The above four parametrizations we have adopted are directly related to the data of type Ia supernovae. Our approach is a useful tool to reexamine the tachyon model. In the future, more accurate observations could help greatly to determine the parameters in the dark energy parametrization. By mapping of the recent expansion history, we will learn more about the essence of the dark energy.

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[1] A.Riess et al., *Astron.J.* **116**, 1009 (1998).

[2] D.N.Spergel et al., *Astrophys.J.Suppl.Ser.* **148**, 175 (2003).

[3] C.Wetterich, *Nucl.Phys.B* **302**, 668 (1988).

[4] B.Ratra and J.Peebles, *Phys.Rev.D* **37**, 321 (1988).

[5] A.Y.Kamenshchick, U.Moschella and V.Pasquier, *Phys.Lett.B* **511**, 265 (2001).

[6] M.C.Bento, O.Bertolanci and A.A.Sen, *Phys.Rev.D* **66**, 043507 (2002).

[7] K.Freese and M.Lewis, *Phys.Lett.B* **540**, 1 (2002).

[8] R.R.Caldwell, M.Kamionkowski and N.N.Weinberg, *Phys.Rev.Lett.* **91**, 071301 (2003).

[9] C.Armendáriz-Picón, T.Damour and V.Mukhanov, *Phys.Lett.B* **458**, 209 (1999).

[10] Z.K.Guo, N.Ohtab and Y.Z.Zhang, *Phys.Rev.D* **72**, 023504 (2005).

[11] P.S.Corasaniti and E.J.Copeland, *Phys.Rev.D* **67**, 063521 (2003).

[12] T.Padmanabhan, *Phys.Rev.D* **66**, 021301 (2002).

[13] T.Padmanabhan and T.R.Choudhury, *Phys.Rev.D* **66**, 081301 (2002).

[14] A.Sen, *JHEP* **10**, 008 (1999).

[15] M.R.Garousi, *Nucl.Phys.B* **584**, 284 (2000).

[16] A.Sen, *JHEP* **0204**, 048 (2002).

[17] A.Sen, *JHEP* **0207**, 065 (2002).

[18] G.W.Gibbons, *Phys.Lett.B* **537**, 1 (2002).

[19] S.Nojiri and S.D.Odintsov, *Phys.Lett.B* **571**, 1 (2003).

[20] B.C.Paul and M.Sami, *Phys.Rev.D* **70**, 027301 (2004).

[21] G.Calcagni, *Phys.Rev.D* **70**, 103525 (2004).

[22] J.S.Bagla, H.K.Jassal and T.Padmanabhan, *Phys.Rev.D* **67**, 063504 (2003).

[23] L.R.W.Abramo and F.Finelli, *Phys.Lett.B* **575**, 165 (2003).

[24] Z.K.Guo and Y.Z.Zhang, *JCAP* **0408**, 010 (2004).

[25] E.J.Copeland, M.R.Garousi, M.Sami and S.Tsujikawa, *Phys.Rev.D* **71**, 043003 (2005).

[26] S.Hannestad and E.Mortsell, *Phys.Rev.D* **66**, 063508 (2002).

[27] A.R.Cooray and D.Huterer, *Astrophys.J.* **513**, L95 (1999).

[28] E.V.Linder, *Phys.Rev.Lett.* **90**, 091301 (2003).

[29] T.Padmanabhan and T.R.Choudhury, *Mon.Not.R.Astron.Soc.* **344**, 823 (2003).

[30] B.F.Gerke and G.Efstathiou, *Mon.Not.R.Astron.Soc.* **335**, 33 (2002).